JHEP HYPER VERSION

Received: *April 5, 2000*, Accepted: *May 26, 2000*

3-point functions of universal scalars in maximal SCFTs at large N

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Abstract: We compute all 3-point functions of the "universal" scalar operators contained in the interacting, maximally supersymmetric CFTs at large N by using the AdS/CFT correspondence. These SCFTs are related to the low energy description of M5, M2 and D3-branes, and the common set of universal scalars corresponds through the AdS/CFT relation to the fluctuations of the metric and the magnetic potential along the internal manifold. For the interacting $(0, 2)$ $SCFT_6$ at large N, which is related to M5-branes, this set of scalars is complete, while additional non-universal scalar operators are present in the $d = 4$, $\mathcal{N} = 4$ super Yang-Mills theory and in the $\mathcal{N} = 8$ $SCFT_3$, related to D3 and M2-branes, respectively.

Keywords: Field Theories in Higher Dimensions, 1/N Expansion, M-Theory, D-branes.

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1. Introduction

The low energy dynamics of non-dilatonic superstring/M-theory branes identify an interesting class of interacting CFT possessing maximal supersymmetry [1]. These SCFT should presumably be given a description in terms of the collective coordinates of the branes. This is well known for a system of N coinciding D3-branes, whose collective degrees of freedom span the $d = 4$, $\mathcal{N} = 4$ $\mathrm{U}(N)$ super vector multiplet and whose infrared dynamics is described precisely by the corresponding super Yang-Mills theory [2]. A similar explicit description for a set of N coinciding M5 or M2-branes is unknown. Indeed, it is only known that the collective coordinates of a single M5-brane form a $d = 6$, $\mathcal{N} = (0, 2)$ free tensor multiplet (a 2-form with selfdual field strength, 5 scalars and 2 Weyl fermions) [3, 4] and that those of a single M2-brane form a $d = 3$, $\mathcal{N} = 8$ free scalar multiplet (8 bosons and 8 Majorana fermions) [5]. Interacting SCFTs describing the collective coordinates of N coinciding M5 or M2-branes remain instead quite mysterious. However, superstring/M-theory predicts the existence of these models [6, 7, 8] and, in fact, suggests a full ADE classification. A concrete handle on the problem is provided by the AdS/CFT duality conjecture, which relates these SCFT in $d = 3, 4, 6$ to superstring/M-theory on $AdS_{4,5,7} \times S_{7,5,4}$ [9, 10, 11]. Specifically, at large N, one can approximate the superstring/M-theory by the corresponding classical supergravity. Then, the latter may be used to obtain

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informations on the strong coupling limit of the related interacting SCFTs and, in the case of $SCFT_{3,6}$, also to get important clues about their mysterious lagrangian formulation. The AdS/CFT duality conjecture has been tested extensively in the literature (see ref. [12] and references therein) and used to compute 3- and 4-point functions of some chiral operators (a partial list consists of refs. [13]–[24]).

In this paper, we continue our analysis of the AdS/CFT correspondence presented in [22, 23, 24], and address the problem of computing the 3-point functions for a set of scalar operators which are present in all of the above mentioned theories at large N. In fact, there are three families of such scalars, to be denoted by $\mathcal{O}^s$, $\mathcal{O}^f$, $\mathcal{O}^t$, which are in correspondence with the metric and the magnetic potential fluctuations along the internal manifold (the metric contributing with two Kaluza-Klein families due to its splitting into a trace and a traceless part). They form a kind of "universal" scalar sector common to all the non-dilatonic branes, which is reminiscent of the NS-NS universal sector present in the spectrum of the various closed superstrings. At large N, all but a finite number of the supersymmetric short multiplets contain precisely one scalar $\mathcal{O}^s$, $\mathcal{O}^f$, $\mathcal{O}^t$, with $\mathcal{O}^s$ being the chiral primary. Their conformal dimensions are given by

$$\Delta^s = \Delta\,, \qquad \Delta^f = \Delta + 2\,, \qquad \Delta^t = \Delta + 4\,, \tag{1.1}$$

where

$$\Delta = \frac{n+1}{D-n-3}\,k\,, \tag{1.2}$$

n is the dimension of the brane, D is the space time dimension and $k \geq 4$ is an integer characterizing the multiplet. The exceptional multiplets, corresponding to the values $k = 2, 3$, do not contain the operators $\mathcal{O}_f$ and $\mathcal{O}_t$. For the M5-brane case, this set of scalars is complete at large N, as all other operators have a non-trivial tensorial character. On the other hand, for the M2 and D3-branes, additional non-universal scalars are present in the spectrum (see, e.g. the tables in [25, 26]).

The simultaneous treatment of the universal sector is made possible by the use of the general gravitational model introduced in [23], which was shown later in [24] to correctly reproduce the universal scalar self couplings also in the case of the dyonic D3-brane (where the full lagrangian of type-IIB supergravity contains a 4-form with selfdual field strength).

Thus, in the following, we briefly review our general gravitational model and present the result for all of the 3-point couplings of the universal scalars. As a check, we have carried out the computation in two independent ways, by expanding the action at the cubic order in the scalar fields and by studying the quadratic corrections to the scalar equations of motion. Of course, we obtained the same final result. Then, the application of AdS/CFT duality allows us to obtain the announced universal scalar 3-point functions. We tried to cast the resulting expressions in a

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way which may suggest a group theoretic interpretation. Finally, we present some technical details on tensor spherical harmonics and their integrals in appendix A.

2. Identification of the universal scalar sector

According to the AdS/CFT duality principles [9, 10, 11], the low energy world volume CFT of N coincident M5, M2, D3-branes at large N is described by $D = 11$, $D = 11$, $D = 10$ type-IIB classical supergravity compactified on $AdS_7 \times S_4$, $AdS_4 \times S_7$, $AdS_5 \times S_5$, respectively. The CFT scalar fields, whose 3-point functions we want to compute, are related by duality in a precise way to certain fluctuations of the AdS bosonic supergravity fields around a maximally supersymmetric Freund-Rubin background [27]. In refs. [23, 24], it has been shown that the dynamics of these fluctuations up to third order is governed by a gravitational action that has the same form for all the three types of branes mentioned above and is, therefore, universal. Let us describe and justify the model briefly.

Space time M_D is $AdS_{D-2-p} \times S_{2+p}$. The relevant fields are the metric g and the $1+p$ form field A_{1+p} with field strength $F_{2+p} = dA_{1+p}$.[1] The Freund-Rubin background $\bar{g}$, $\bar{A}_{1+p}$ is such that $\bar{g}$ is factorized and $\bar{F}_{2+p} = \bar{F}_{(0,2+p)}$. The relevant bosonic fluctuations of g and A_{1+p} around the background are such that g remains factorized and $F_{2+p} = \bar{F}_{(0,2+p)} + da_{(0,1+p)}$.[2] The action is given by

$$I = \frac{1}{4\kappa^2} \int_{AdS_{D-2-p} \times S_{2+p}} \Big[R(g) *_g 1 - F_{2+p} \wedge *_g F_{2+p} \Big] , \tag{2.1}$$

$$g = g' \oplus g'' , \qquad F_{2+p} = \bar{F}_{(0,2+p)} + da_{(0,1+p)} . \tag{2.2}$$

The important cases are those for which $(D, p) = (11, 2), (11, 5), (10, 3)$.

For the M5 theory ($(D, p) = (11, 2)$), the above action is obtained directly from that of the bosonic sector of $D = 11$ supergravity by noting that, for the fluctuation considered here, the Chern-Simons term vanishes identically.

For the M2 theory ($(D, p) = (11, 5)$), the situation is slightly more complicated. Using the standard 3-form formulation of $D = 11$ supergravity is inconvenient as the relevant scalar fluctuation contained in the fluctuation $a_{(3,0)}$ of A_3 comes about as the solution of the constraint

$$d *_{\bar{g}'} a_{(3,0)} = 0 \tag{2.3}$$

entailed by gauge fixing at quadratic level, which is difficult to implement in an off-shell fashion. This problem can be solved by means of a standard dualization trick,

[1] We denote form degree on M_D by a subfix, e.g. ω_r is an r form on M_D.

[2] We say that a metric g on M_D is factorized if g has the block structure $g = g' \oplus g''$, where $\bar{g}'$, $\bar{g}''$ are metrics on AdS_{D-2-p}, S_{2+p}, respectively. Similarly, we denote form degree on the factors AdS_{D-2-p}, S_{2+p} by a pair of suffixes, e.g. $\omega_{(r,s)}$ denotes an $r+s$ form on M_D that is an r form on AdS_{D-2-p} and an s form on S_{2+p}.

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whereby the 3-form A_3 is replaced by a 6-form A_6 such that $F_7 = *_g F_4$. Since the Chern-Simons term vanishes also in this case for the fluctuation considered here, the resulting action takes the simple form (2.1). However, in the dual formulation, the relevant scalar fluctuation is contained in the fluctuation $a_{(0,6)}$ of A_6 and can thus be treated in an off-shell fashion in a way completely analogous to that of the M5-brane.

For the D3 theory ($(D,p) = (10,3)$), the relevant fields are the metric g and the IIB Ramond-Ramond 4-form field A_4 with selfdual field strength $F_5^{\rm sd} = dA_4$

$$F_5^{\rm sd} = *_g F_5^{\rm sd} . \tag{2.4}$$

The selfdual Freund-Rubin background $\bar{g}$, $\bar{A}_4$ is such that $\bar{g}$ is factorized and $\bar{F}_5^{\rm sd} = 2^{-1/2}(\bar{F}_{(0,5)} + *_{\bar{g}} \bar{F}_{(0,5)})$. The relevant fluctuations of g and A_4 around the background are such that g is factorized, as usual, and $F_5^{\rm sd} = \bar{F}_5^{\rm sd} + da_4$, where $a_4 = 2^{-1/2}(a_{(4,0)} + a_{(0,4)})$. The selfduality equations (2.4) relate the fluctuations $a_{(4,0)}$, $a_{(0,4)}$ and allow in principle to express $a_{(4,0)}$ in terms of $a_{(0,4)}$. Upon doing this, the resulting field equations can be seen to be equivalent to those obtained from the action (2.1).

The standard $AdS_{D-2-p} \times S_{2+p}$ Freund Rubin solution $\bar{g}_{ij}$, $\bar{A}_{i_1 \cdots i_{1+p}}$ of the field equations following from the action (2.1) is given by:

$$\bar{R}_{\kappa\lambda\mu\nu} = -a^2 \Big(\bar{g}_{\kappa\mu}\, \bar{g}_{\lambda\nu} - \bar{g}_{\kappa\nu}\, \bar{g}_{\lambda\mu} \Big) , \qquad a^2 = \frac{(1+p)}{(D-2)(D-3-p)}\, e^2 , \tag{2.5a}$$

$$\bar{R}_{\alpha\beta\gamma\delta} = \bar{e}^2 \Big(\bar{g}_{\alpha\gamma}\, \bar{g}_{\beta\delta} - \bar{g}_{\alpha\delta}\, \bar{g}_{\beta\gamma} \Big) , \qquad \bar{e}^2 = \frac{(D-3-p)}{(D-2)(1+p)}\, e^2 , \tag{2.5b}$$

$$\bar{F}_{\alpha_1 \cdots \alpha_{2+p}} = e\, \bar{\epsilon}_{\alpha_1 \cdots \alpha_{2+p}} , \tag{2.6}$$

where $\bar{\epsilon}_{\alpha_1 \cdots \alpha_{2+p}}$ denotes the standard volume form on the unit sphere and e is an arbitrary mass scale parameterizing the compactification.[3] The other components of the Riemann tensor and the field strength vanish identically.

We expand the action in fluctuations around the background $\bar{g}_{ij}$, $\bar{A}_{i_1 \cdots i_{1+p}}$. We parameterize the fluctuations δg_{ij}, $\delta A_{i_1 \cdots i_{1+p}}$ of the fields g_{ij}, $A_{i_1 \cdots i_{1+p}}$ around the background as in [28]

$$\delta g_{\alpha\beta} = f_{\alpha\beta} + \bar{\nabla}_\alpha n_\beta + \bar{\nabla}_\beta n_\alpha + \left(\bar{\nabla}_\alpha \bar{\nabla}_\beta - \frac{1}{2+p}\, \bar{g}_{\alpha\beta}\, \bar{\nabla}^\gamma \bar{\nabla}_\gamma \right) q + \frac{1}{2+p}\, \bar{g}_{\alpha\beta}\, \pi ,$$
$$f^\gamma{}_\gamma = 0 , \qquad \bar{\nabla}^\gamma f_{\gamma\alpha} = 0 , \qquad \bar{\nabla}^\gamma n_\gamma = 0 , \tag{2.7}$$

$$\delta A_{\alpha_1 \cdots \alpha_{1+p}} = (1+p) \bar{\nabla}_{[\alpha_1} a_{\alpha_2 \cdots \alpha_{1+p}]} + \bar{\epsilon}_{\alpha_1 \cdots \alpha_{1+p}}{}^\gamma\, \bar{\nabla}_\gamma b ,$$
$$\bar{\nabla}^\gamma\, a_{\gamma\alpha_3 \cdots \alpha_{1+p}} = 0 . \tag{2.8}$$

[3]In this paper, we adopt the following conventions. Latin lower case letters $i, j, k, l, \ldots$ denote M_D indices. Late Greek lower case letters $\kappa, \lambda, \mu, \nu, \ldots$ denote AdS_{D-2-p} indices. Early Greek lower case letters $\alpha, \beta, \gamma, \delta, \ldots$ denote S_{2+p} indices.

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Fluctuations of the other components of g_{ij} and $A_{i_1\cdots i_{1+p}}$ can be disregarded as they are independent up to third order from the ones we are interested in and therefore do not contribute to the relevant scalar 3-point couplings.

The gauge can be partially fixed by eliminating those gauge invariances which do not correspond to the usual reparameterization and form gauge invariances from the AdS_{D-2-p} perspective. This can be done by imposing

$$\bar{\nabla}^{\beta}\left(\delta g_{\beta\alpha} - \frac{1}{2+p}\,\bar{g}_{\beta\alpha}\,\delta g^{\gamma}{}_{\gamma}\right) = 0\,, \tag{2.9}$$

$$\bar{\nabla}^{\beta}\delta A_{i_1\cdots i_p\beta} = 0 \tag{2.10}$$

as shown in [28]. Fixing the gauge entails a number of constraints which must be disposed of as explained in [24].

There are three universal families of AdS_{D-2-p} scalar fields contained in the fluctuations listed above, which we denote as f_I, s_I, t_I. The scalar fields f_I are defined by expanding $f_{\alpha\beta}$ (cf. eq. (2.7)) with respect to an orthonormal basis $\{Y_I^{(2)}\}$ of symmetric traceless transversal 2-tensor spherical harmonics of S_{2+p} (cf. appendix A)

$$f_{\alpha\beta} = \sum_I f_I\, Y^{(2)}_{I\alpha\beta}\,. \tag{2.11}$$

The scalar fields s_I, t_I are given by linear functionals of π, b (cf. eqs. (2.7) and (2.8)) non local in S_{2+p} defined as follows. One expands the scalar fields π, b with respect to an orthonormal basis $\{Y_I^{(0)}\}$ of scalar spherical harmonics of S_{2+p} (cf. appendix A)

$$\pi = \sum_I \pi_I\, Y_I^{(0)}\,, \tag{2.12a}$$

$$b = \sum_I b_I\, Y_I^{(0)} \tag{2.12b}$$

and identifies the scalar mass eigenstates as given by [23, 24]

$$s_I = \frac{1}{2k+1+p}\left(\frac{1}{2(2+p)(D-3-p)}\,\pi_I + \frac{(-1)^p(k+1+p)}{(1+p)(D-2)}\,e\,b_I\right), \tag{2.13a}$$

$$t_I = \frac{1}{2k+1+p}\left(\frac{1}{2(2+p)(D-3-p)}\,\pi_I - \frac{(-1)^p\,k}{(1+p)(D-2)}\,e\,b_I\right). \tag{2.13b}$$

It should be kept in mind that the range of the quantum numbers I of f_I differs from that of the quantum numbers I of s_I, t_I, as these ranges parameterize orthogonal bases of spherical harmonics of different tensorial rank. However, we shall use the same notation for these different quantum numbers for simplicity, as no confusion is possible.

3. The cubic action of the universal scalar sector

From the action (2.1), one can extract the couplings of the scalars f_I, s_I and t_I defined in the previous section. After performing some field redefinitions, one finds that their action to cubic order is given by

$$I^{fst}_{[\leq 3]} = \frac{1}{4\kappa^2} \int_{AdS_{d+1}} d^{d+1}\, y \, (-\bar{g}_{d+1})^{1/2} \left[\frac{1}{2} \sum_i A_i \, \psi_i \left(\Box - {m_i}^2 \right) \psi_i + \right.$$
$$\left. + \frac{1}{3!} \sum_{ijk} G_{ijk} \, \psi_i \, \psi_j \, \psi_k \right] , \tag{3.1}$$

where $d = D-3-p$, $\Box$ denotes the d'Alembertian on AdS_{d+1} and the index $i = \{I, a\}$ contains a flavor index a for the (f, s, t) types of fields, i.e. $\psi_i = \psi_I^a = (f_I, s_I, t_I)$. The various constants appearing in the actions are given by the following expressions:

$$A_I^f = \frac{1}{2} \, z_I \, \bar{e}^{-2-p} \, , \tag{3.2a}$$
$$A_I^s = \frac{2\nu \, k(k-1)(2k+1+p)}{k+\gamma_s} \, z_I \, \bar{e}^{-2-p} \, , \tag{3.2b}$$
$$A_I^t = \frac{2\nu \, (k+1+p)(k+2+p)(2k+1+p)}{k+\gamma_t} \, z_I \, \bar{e}^{-2-p} \, ; \tag{3.2c}$$

$$m_I^{f\,2} = k \, (k+1+p) \, \bar{e}^2 \, , \tag{3.3a}$$
$$m_I^{s\,2} = k \, (k-1-p) \, \bar{e}^2 \, , \tag{3.3b}$$
$$m_I^{t\,2} = (k+1+p)(k+2+2p) \, \bar{e}^2 \, ; \tag{3.3c}$$

$$G^{fff}_{I_1 I_2 I_3} = \left(\alpha + \frac{1}{2} \, (1+p) \right) a_{I_1 I_2 I_3} \, \langle \mathcal{T}_{I_1} \mathcal{T}_{I_2} \mathcal{T}_{I_3} \rangle \, \bar{e}^{-p} \, , \tag{3.4a}$$

$$G^{ffs}_{I_1 I_2 I_3} = 4(D-2) \, \frac{\alpha_1 \, \alpha_2 \left(\alpha_3 + \frac{1}{2} \, (1+p) \right) \left(\alpha + \frac{1}{2} \, (1+p) \right)}{k_3 + \gamma_s} \times$$
$$\times \, a_{I_1 I_2 I_3} \, \langle \mathcal{T}_{I_1} \mathcal{T}_{I_2} \mathcal{C}_{I_3} \rangle \, \bar{e}^{-p} \, , \tag{3.4b}$$

$$G^{fft}_{I_1 I_2 I_3} = 4(D-2) \, \frac{\left(\alpha_1 + \frac{1}{2}(1+p) \right) \left(\alpha_2 + \frac{1}{2} \, (1+p) \right) \alpha_3 (\alpha + 1 + p)}{k_3 + \gamma_t} \times$$
$$\times \, a_{I_1 I_2 I_3} \, \langle \mathcal{T}_{I_1} \mathcal{T}_{I_2} \mathcal{C}_{I_3} \rangle \, \bar{e}^{-p} \, , \tag{3.4c}$$

$$G^{fss}_{I_1 I_2 I_3} = 8\nu \, \frac{\alpha_1 \left(\alpha_1 - \frac{1}{2}(1+p) \right) \alpha \left(\alpha + \frac{1}{2} \, (1+p) \right)}{(k_2 + \gamma_s)(k_3 + \gamma_s)} \times$$
$$\times \left\{ (\alpha_1 - 1) \left(\alpha + \frac{1+p}{D-3-p} \right) + \frac{\theta}{\nu} \, F^{fss} \right\} a_{I_1 I_2 I_3} \langle \mathcal{T}_{I_1} \mathcal{C}_{I_2} \, \mathcal{C}_{I_3} \rangle \bar{e}^{-p} \, , \tag{3.4d}$$

$$G^{ftt}_{I_1 I_2 I_3} = 8\nu \, \frac{\alpha_1 (\alpha_1 + 1 + p) \left(\alpha + \frac{1}{2} \, (1+p) \right) \left(\alpha + \frac{3}{2} \, (1+p) \right)}{(k_2 + \gamma_t)(k_3 + \gamma_t)} \times \tag{3.4e}$$
$$\times \left\{ \left(\alpha_1 + \frac{(1+p)(D-4-p)}{D-3-p} \right) (\alpha + p + 2) + \frac{\theta}{\nu} \, F^{ftt} \right\} a_{I_1 I_2 I_3} \langle \mathcal{T}_{I_1} \mathcal{C}_{I_2} \, \mathcal{C}_{I_3} \rangle \, \bar{e}^{-p} \, ,$$

$$G^{fst}_{I_1I_2I_3} = 8\nu\, \frac{\alpha_1(\alpha_2+1+p)\left(\alpha_3-\frac{1}{2}(1+p)\right)\left(\alpha+\frac{1}{2}(1+p)\right)}{(k_2+\gamma_s)(k_3+\gamma_t)} \times \tag{3.4f}$$

$$\times\left\{\left(\alpha_2+\frac{(1+p)(D-4-p)}{D-3-p}\right)(\alpha_3-1)+\frac{\theta}{\nu}F^{fst}\right\} a_{I_1I_2I_3}\langle \mathcal{T}_{I_1}\mathcal{C}_{I_2}\,\mathcal{C}_{I_3}\rangle \bar{e}^{-p}\,,$$

$$G^{sss}_{I_1I_2I_3} = 32(D-2)\,\nu\, \frac{\alpha_1\,\alpha_2\,\alpha_3\left(\alpha-\frac{1}{2}(1+p)\right)\left(\alpha+\frac{1}{2}(1+p)\right)}{(k_1+\gamma_s)(k_2+\gamma_s)(k_3+\gamma_s)} \times$$

$$\times\left\{(\alpha-1)\left(\alpha+\frac{1+p}{D-3-p}\right)\left(\alpha+\frac{(1+p)(-D+4+2p)}{2(D-2)}\right)+\frac{\theta}{\nu}\,F^{sss}\right\}\times$$

$$\times\, a_{I_1I_2I_3}\,\langle \mathcal{C}_{I_1}\mathcal{C}_{I_2}\,\mathcal{C}_{I_3}\rangle\,\bar{e}^{-p}\,, \tag{3.4g}$$

$$G^{sst}_{I_1I_2I_3} = 32(D-2)\,\nu\,\times$$

$$\times\,\frac{\left(\alpha_1+\frac{1}{2}(1+p)\right)\left(\alpha_2+\frac{1}{2}(1+p)\right)\alpha_3(\alpha_3-1-p)\left(\alpha+\frac{1}{2}(1+p)\right)}{(k_1+\gamma_s)(k_2+\gamma_s)(k_3+\gamma_t)}\times$$

$$\times\left\{(\alpha_3-1)\left(\alpha_3+\frac{(1+p)(-D+3+p)}{D-2}\right)\times\right. \tag{3.4h}$$

$$\left.\times\,\left(\alpha_3+\frac{(1+p)(-D+4+p)}{D-3-p}\right)-\frac{\theta}{\nu}F^{sst}\right\}a_{I_1I_2I_3}\,\langle \mathcal{C}_{I_1}\mathcal{C}_{I_2}\,\mathcal{C}_{I_3}\rangle\,\bar{e}^{-p}\,,$$

$$G^{tts}_{I_1I_2I_3} = 32(D-2)\,\nu\;\frac{\alpha_1\,\alpha_2\left(\alpha_3+\frac{1}{2}(1+p)\right)\left(\alpha_3+\frac{3}{2}(1+p)\right)(\alpha+1+p)}{(k_1+\gamma_t)(k_2+\gamma_t)(k_3+\gamma_s)}\times$$

$$\times\left\{(\alpha_3+2+p)\left(\alpha_3+\frac{(1+p)(3D-8-2p)}{2(D-2)}\right)\times\right.$$

$$\left.\times\,\left(\alpha_3+\frac{(1+p)(D-4-p)}{D-3-p}\right)-\frac{\theta}{\nu}\,F^{tts}\right\}a_{I_1I_2I_3}\,\langle \mathcal{C}_{I_1}\mathcal{C}_{I_2}\,\mathcal{C}_{I_3}\rangle\,\bar{e}^{-p}\,, \tag{3.4i}$$

$$G^{ttt}_{I_1I_2I_3} = 32(D-2)\,\nu\,\times$$

$$\times\,\frac{\left(\alpha_1+\frac{1}{2}(1+p)\right)\left(\alpha_2+\frac{1}{2}(1+p)\right)\left(\alpha_3+\frac{1}{2}(1+p)\right)(\alpha+1+p)(\alpha+2+2p)}{(k_1+\gamma_t)(k_2+\gamma_t)(k_3+\gamma_t)}\times$$

$$\times\,\left\{(\alpha+2+p)\left(\alpha+\frac{(1+p)(2D-8-2p)}{D-3-p}\right)\times\right.$$

$$\left.\times\left(\alpha+\frac{(1+p)(4D-9-p)}{2(D-2)}\right)+\frac{\theta}{\nu}\,F^{ttt}\right\}a_{I_1I_2I_3}\,\langle \mathcal{C}_{I_1}\mathcal{C}_{I_2}\,\mathcal{C}_{I_3}\rangle\,\bar{e}^{-p}\,, \tag{3.4j}$$

where we have defined for notational convenience

$$\nu = (D-2)(1+p)(D-3-p)\,, \tag{3.5a}$$

$$\gamma_s = \frac{1+p}{D-3-p}\,,\qquad \gamma_t = \frac{(1+p)(D-4-p)}{D-3-p}\,, \tag{3.5b}$$

$$\theta = (1+p)(D-3-p)-2(D-2)\,, \tag{3.5c}$$

$\bar{e}^2$ is defined in eq. (2.5b), while α_1, α_2, α_3, α, z_I, $a_{I_1I_2I_3}$ and the contractions $\langle\mathcal{TTT}\rangle$, $\langle\mathcal{TTC}\rangle$, $\langle\mathcal{TCC}\rangle$, $\langle\mathcal{CCC}\rangle$ are defined in appendix A. Note that the auxiliary functions $F^{fss}, F^{ftt}, \ldots$ appearing in the couplings are always multiplied by θ and do not contribute to the relevant cases of $AdS_{5,7,4} \times S_{5,4,7}$ where θ vanishes. The explicit expressions for such functions, which may be needed in future applications, are listed in appendix B.

Writing above

$$A_I = \bar{A}_I\, z_I\, \bar{e}^{-2-p}\,, \tag{3.6}$$

$$m_I{}^2 = \bar{m}_I{}^2\, \bar{e}^2\,, \tag{3.7}$$

$$G_{I_1I_2I_3} = \bar{G}_{I_1I_2I_3}\, a_{I_1I_2I_3}\, \langle\mathcal{Y}_{I_1}\,\mathcal{Y}_{I_2}\,\mathcal{Y}_{I_3}\rangle\, \bar{e}^{-p}\,, \tag{3.8}$$

where the $\langle\mathcal{Y}_{I_1}\mathcal{Y}_{I_2}\mathcal{Y}_{I_3}\rangle$ denotes the appropriate tensor/scalar harmonic contractions, we get the following expressions:

$AdS_7 \times S_4$.

$$\bar{A}^f_I = \frac{1}{2}\,, \tag{3.9a}$$

$$\bar{A}^s_I = \frac{2^2\,3^4\,k(k-1)(2k+3)}{k+\frac{1}{2}}\,, \tag{3.9b}$$

$$\bar{A}^t_I = \frac{2^2\,3^4\,(k+3)(k+4)(2k+3)}{k+\frac{5}{2}}\,; \tag{3.9c}$$

$$\bar{m}^{f\,2}_I = k(k+3)\,, \tag{3.10a}$$

$$\bar{m}^{s\,2}_I = k(k-3)\,, \tag{3.10b}$$

$$\bar{m}^{t\,2}_I = (k+3)(k+6)\,; \tag{3.10c}$$

$$\bar{G}^{fff}_{I_1I_2I_3} = \alpha + \frac{3}{2}\,, \tag{3.11a}$$

$$\bar{G}^{ffs}_{I_1I_2I_3} = \frac{2^2\,3^2\;\alpha_1\,\alpha_2\left(\alpha_3+\frac{3}{2}\right)\left(\alpha+\frac{3}{2}\right)}{k_3+\frac{1}{2}}\,, \tag{3.11b}$$

$$\bar{G}^{fft}_{I_1I_2I_3} = \frac{2^2\,3^2\left(\alpha_1+\frac{3}{2}\right)\left(\alpha_2+\frac{3}{2}\right)\alpha_3(\alpha+3)}{k_3+\frac{5}{2}}\,, \tag{3.11c}$$

$$\bar{G}^{fss}_{I_1I_2I_3} = \frac{2^4\,3^4\left(\alpha_1-\frac{3}{2}\right)(\alpha_1-1)\alpha_1\alpha\left(\alpha+\frac{1}{2}\right)\left(\alpha+\frac{3}{2}\right)}{\left(k_2+\frac{1}{2}\right)\left(k_3+\frac{1}{2}\right)}\,, \tag{3.11d}$$

$$\bar{G}^{ftt}_{I_1I_2I_3} = \frac{2^4\,3^4\;\alpha_1\left(\alpha_1+\frac{5}{2}\right)(\alpha_1+3)\left(\alpha+\frac{3}{2}\right)(\alpha+4)\left(\alpha+\frac{9}{2}\right)}{\left(k_2+\frac{5}{2}\right)\left(k_3+\frac{5}{2}\right)}\,, \tag{3.11e}$$

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$$\bar{G}^{fst}_{I_1I_2I_3} = \frac{2^4\,3^4\,\alpha_1\left(\alpha_2+\frac{5}{2}\right)(\alpha_2+3)(\alpha_3-1)\left(\alpha_3-\frac{3}{2}\right)\left(\alpha+\frac{3}{2}\right)}{\left(k_2+\frac{1}{2}\right)\left(k_3+\frac{5}{2}\right)}\,, \tag{3.11f}$$

$$\bar{G}^{sss}_{I_1I_2I_3} = \frac{2^6\,3^6\,\alpha_1\,\alpha_2\,\alpha_3\,(\alpha-1)\left(\alpha^2-\frac{1}{4}\right)\left(\alpha^2-\frac{9}{4}\right)}{\left(k_1+\frac{1}{2}\right)\left(k_2+\frac{1}{2}\right)\left(k_3+\frac{1}{2}\right)}\,, \tag{3.11g}$$

$$\bar{G}^{sst}_{I_1I_2I_3} = \frac{2^6\,3^6\left(\alpha_1+\frac{3}{2}\right)\left(\alpha_2+\frac{3}{2}\right)\alpha_3(\alpha_3-1)(\alpha_3-2)\left(\alpha_3-\frac{5}{2}\right)(\alpha_3-3)\left(\alpha+\frac{3}{2}\right)}{\left(k_1+\frac{1}{2}\right)\left(k_2+\frac{1}{2}\right)\left(k_3+\frac{5}{2}\right)}\,, \tag{3.11h}$$

$$\bar{G}^{tts}_{I_1I_2I_3} = \frac{2^6\,3^6\,\alpha_1\,\alpha_2\left(\alpha_3+\frac{3}{2}\right)\left(\alpha_3+\frac{5}{2}\right)\left(\alpha_3+\frac{7}{2}\right)(\alpha_3+4)\left(\alpha_3+\frac{9}{2}\right)(\alpha+3)}{\left(k_1+\frac{5}{2}\right)\left(k_2+\frac{5}{2}\right)\left(k_3+\frac{1}{2}\right)}\,, \tag{3.11i}$$

$$\bar{G}^{ttt}_{I_1I_2I_3} = \frac{2^6\,3^6\left(\alpha_1+\frac{3}{2}\right)\left(\alpha_2+\frac{3}{2}\right)\left(\alpha_3+\frac{3}{2}\right)(\alpha+3)(\alpha+4)(\alpha+5)\left(\alpha+\frac{11}{2}\right)(\alpha+6)}{\left(k_1+\frac{5}{2}\right)\left(k_2+\frac{5}{2}\right)\left(k_3+\frac{5}{2}\right)}\,. \tag{3.11j}$$

$AdS_4 \times S_7$.

$$\bar{A}^f_I = \frac{1}{2}\,, \tag{3.12a}$$

$$\bar{A}^s_I = \frac{2^2\,3^4\,k(k-1)(2k+6)}{k+2}\,, \tag{3.12b}$$

$$\bar{A}^t_I = \frac{2^2\,3^4\,(k+6)(k+7)(2k+6)}{k+4}\,; \tag{3.12c}$$

$$\bar{m}^{f\,2}_I = k(k+6)\,, \tag{3.13a}$$

$$\bar{m}^{s\,2}_I = k(k-6)\,, \tag{3.13b}$$

$$\bar{m}^{t\,2}_I = (k+6)(k+12)\,; \tag{3.13c}$$

$$\bar{G}^{fff}_{I_1I_2I_3} = \alpha+3\,, \tag{3.14a}$$

$$\bar{G}^{ffs}_{I_1I_2I_3} = \frac{2^2\,3^2\,\alpha_1\,\alpha_2\,(\alpha_3+3)(\alpha+3)}{k_3+2}\,, \tag{3.14b}$$

$$\bar{G}^{fft}_{I_1I_2I_3} = \frac{2^2\,3^2\,(\alpha_1+3)(\alpha_2+3)\alpha_3(\alpha+6)}{k_3+4}\,, \tag{3.14c}$$

$$\bar{G}^{fss}_{I_1I_2I_3} = \frac{2^4\,3^4\,(\alpha_1-3)(\alpha_1-1)\alpha_1\alpha(\alpha+2)(\alpha+3)}{(k_2+2)(k_3+2)}\,, \tag{3.14d}$$

$$\bar{G}^{ftt}_{I_1I_2I_3} = \frac{2^4\,3^4\,\alpha_1\,(\alpha_1+4)(\alpha_1+6)(\alpha+3)(\alpha+7)(\alpha+9)}{(k_2+4)(k_3+4)}\,, \tag{3.14e}$$

$$\bar{G}^{fst}_{I_1I_2I_3} = \frac{2^4\,3^4\,\alpha_1\,(\alpha_2+4)(\alpha_2+6)(\alpha_3-3)(\alpha_3-1)(\alpha+3)}{(k_2+2)(k_3+4)}\,, \tag{3.14f}$$

$$\bar{G}^{sss}_{I_1I_2I_3} = \frac{2^6\,3^6\,\alpha_1\,\alpha_2\,\alpha_3(\alpha+2)\,(\alpha^2-1)\,(\alpha^2-9)}{(k_1+2)(k_2+2)(k_3+2)}\,, \tag{3.14g}$$

$$\bar{G}^{sst}_{I_1I_2I_3} = \frac{2^6\,3^6(\alpha_1+3)(\alpha_2+3)\alpha_3(\alpha_3-1)(\alpha_3-2)(\alpha_3-4)(\alpha_3-6)(\alpha+3)}{(k_1+2)(k_2+2)(k_3+4)}\,, \tag{3.14h}$$

$$\bar{G}^{tts}_{I_1I_2I_3} = \frac{2^6\,3^6\,\alpha_1\,\alpha_2\,(\alpha_3+3)(\alpha_3+4)(\alpha_3+5)(\alpha_3+7)(\alpha_3+9)(\alpha+6)}{(k_1+4)(k_2+4)(k_3+2)}\,, \tag{3.14i}$$

$$\bar{G}^{ttt}_{I_1I_2I_3} = \frac{2^6\,3^6\,(\alpha_1+3)(\alpha_2+3)(\alpha_3+3)(\alpha+6)(\alpha+7)(\alpha+8)(\alpha+10)(\alpha+12)}{(k_1+4)(k_2+4)(k_3+4)}\,. \tag{3.14j}$$

$AdS_5\times S_5$**.**

$$\bar{A}^f_I = \frac{1}{2}\,, \tag{3.15a}$$

$$\bar{A}^s_I = \frac{2^8\,k(k-1)(2k+4)}{k+1}\,, \tag{3.15b}$$

$$\bar{A}^t_I = \frac{2^8\,(k+4)(k+5)(2k+4)}{k+3}\,; \tag{3.15c}$$

$$\bar{m}^{f\,2}_I = k(k+4)\,, \tag{3.16a}$$

$$\bar{m}^{s\,2}_I = k(k-4)\,, \tag{3.16b}$$

$$\bar{m}^{t\,2}_I = (k+4)(k+8)\,; \tag{3.16c}$$

$$\bar{G}^{fff}_{I_1I_2I_3} = \alpha+2\,, \tag{3.17a}$$

$$\bar{G}^{ffs}_{I_1I_2I_3} = \frac{2^5\,\alpha_1\,\alpha_2\,(\alpha_3+2)(\alpha+2)}{k_3+1}\,, \tag{3.17b}$$

$$\bar{G}^{fft}_{I_1I_2I_3} = \frac{2^5\,(\alpha_1+2)(\alpha_2+2)\alpha_3(\alpha+4)}{k_3+3}\,, \tag{3.17c}$$

$$\bar{G}^{fss}_{I_1I_2I_3} = \frac{2^{10}\,(\alpha_1-2)(\alpha_1-1)\alpha_1\alpha(\alpha+1)(\alpha+2)}{(k_2+1)(k_3+1)}\,, \tag{3.17d}$$

$$\bar{G}^{ftt}_{I_1I_2I_3} = \frac{2^{10}\,\alpha_1\,(\alpha_1+3)(\alpha_1+4)(\alpha+2)(\alpha+5)(\alpha+6)}{(k_2+3)(k_3+3)}\,, \tag{3.17e}$$

$$\bar{G}^{fst}_{I_1I_2I_3} = \frac{2^{10}\,\alpha_1\,(\alpha_2+3)(\alpha_2+4)(\alpha_3-2)(\alpha_3-1)(\alpha+2)}{(k_2+1)(k_3+3)}\,, \tag{3.17f}$$

$$\bar{G}^{sss}_{I_1 I_2 I_3} = \frac{2^{15}\, \alpha_1\, \alpha_2\, \alpha_3\, \alpha\, (\alpha^2 - 1)\, (\alpha^2 - 4)}{(k_1 + 1)(k_2 + 1)(k_3 + 1)}\,, \tag{3.17g}$$

$$\bar{G}^{sst}_{I_1 I_2 I_3} = \frac{2^{15}(\alpha_1 + 2)(\alpha_2 + 2)\alpha_3(\alpha_3 - 1)(\alpha_3 - 2)(\alpha_3 - 3)(\alpha_3 - 4)(\alpha + 2)}{(k_1 + 1)(k_2 + 1)(k_3 + 3)}\,, \tag{3.17h}$$

$$\bar{G}^{tts}_{I_1 I_2 I_3} = \frac{2^{15}\alpha_1\alpha_2(\alpha_3 + 2)(\alpha_3 + 3)(\alpha_3 + 4)(\alpha_3 + 5)(\alpha_3 + 6)(\alpha + 4)}{(k_1 + 3)(k_2 + 3)(k_3 + 1)}\,, \tag{3.17i}$$

$$\bar{G}^{ttt}_{I_1 I_2 I_3} = \frac{2^{15}(\alpha_1 + 2)(\alpha_2 + 2)(\alpha_3 + 2)(\alpha + 4)(\alpha + 5)(\alpha + 6)(\alpha + 7)(\alpha + 8)}{(k_1 + 3)(k_2 + 3)(k_3 + 3)}\,. \tag{3.17j}$$

4. Computation of the 3-point functions of the universal scalar sector

We are now ready to compute 2- and 3-point functions in the SCFTs using the AdS_{d+1}/CFT_d correspondence. The general formulas derived in [29, 30] work with AdS radius set to 1. Assume that the AdS scalar fields ϕ_i correspond to the CFT local field $\mathcal{O}_i$. The mass m_i of ψ_i and the conformal dimension Δ_i of $\mathcal{O}_i$ are related as

$$\Delta_i = \frac{1}{2}\left[d + \left(d^2 + 4\, {m_i}^2\right)^{1/2}\right]. \tag{4.1}$$

Then

$$\Big\langle \mathcal{O}_i(x)\, \mathcal{O}_j(y) \Big\rangle = \frac{A_i}{4\kappa^2}\, \frac{2}{\pi^{d/2}}\, \frac{\Delta_i - \frac{d}{2}}{\Delta_i}\, \frac{\Gamma(\Delta_i + 1)}{\Gamma\left(\Delta_i - \frac{d}{2}\right)}\, \frac{(w_i)^2\, \delta_{ij}}{|x - y|^{2\Delta_i}}\,, \tag{4.2}$$

where $A_i/4\kappa^2$ is the coefficient of the canonically normalized kinetic term of the bulk field ψ_i as in eq. (3.1) and

$$\Big\langle \mathcal{O}_i(x)\, \mathcal{O}_j(y)\, \mathcal{O}_k(z) \Big\rangle = \frac{R_{ijk}}{|x - y|^{\Delta_i + \Delta_j - \Delta_k}\, |y - z|^{\Delta_j + \Delta_k - \Delta_i}\, |z - x|^{\Delta_k + \Delta_i - \Delta_j}}\,, \tag{4.3}$$

with

$$\begin{aligned} R_{ijk} = {} & \frac{G_{ijk}}{4\kappa^2}\, \frac{1}{2\pi^d} \times \\ & \times \frac{\Gamma\left(\frac{1}{2}\left(\Delta_i + \Delta_j - \Delta_k\right)\right) \Gamma\left(\frac{1}{2}\left(\Delta_j + \Delta_k - \Delta_i\right)\right) \Gamma\left(\frac{1}{2}\left(\Delta_k + \Delta_i - \Delta_j\right)\right)}{\Gamma\left(\Delta_i - \frac{d}{2}\right) \Gamma\left(\Delta_j - \frac{d}{2}\right) \Gamma\left(\Delta_k - \frac{d}{2}\right)} \times \\ & \times \Gamma\left(\frac{1}{2}\Big(\Delta_i + \Delta_j + \Delta_k - d\Big)\right) w_i\, w_j\, w_k\,, \end{aligned} \tag{4.4}$$

where $G_{ijk}/4\kappa^2$ is the cubic coupling constant of ψ_i, ψ_j, ψ_k as in eq. (3.1). The factors w_i parameterize unknown proportionality constants which relate the fields ψ_i

to the sources of the operators $\mathcal{O}_i$, namely the generating functional of correlators reads as $\langle e^{\int w_i \psi_i \mathcal{O}_i} \rangle_{\text{SCFT}}$. For the present purposes we follow ref. [13] and fix them to normalize the 2-point functions as

$$\Big\langle \mathcal{O}_i(x)\, \mathcal{O}_j(y) \Big\rangle = \frac{\delta_{ij}}{|x-y|^{2\Delta_i}} \,. \tag{4.5}$$

With this canonical normalization the 3-point functions are readily computed.

In our case, $d = D - 3 - p$. Imposing that the AdS radius is 1 fixes the value of $\bar{e}$ to be

$$\bar{e} = \frac{d}{1+p} \,. \tag{4.6}$$

We denote by $\mathcal{O}_I^f$, $\mathcal{O}_I^s$, $\mathcal{O}_I^t$ the CFT_d operators corresponding to the AdS_{d+1} scalars f_I, s_I, t_I in the AdS_{d+1}/CFT_d duality. Their dimensions are given by

$$\Delta_I^f = \frac{d(k+1+p)}{1+p} \,, \tag{4.7a}$$

$$\Delta_I^s = \frac{dk}{1+p} \,, \tag{4.7b}$$

$$\Delta_I^t = \frac{d(k+2+2p)}{1+p} \,. \tag{4.7c}$$

Separating out the group theory factors in the R_{ijk} coefficients by defining

$$R_{ijk} = \bar{R}_{ijk} \, \langle \mathcal{Y}_{I_1} \, \mathcal{Y}_{I_2} \, \mathcal{Y}_{I_3} \rangle \tag{4.8}$$

as in eq. (3.8), we find the following expressions:

$AdS_7 \times S_4$. In this case one has $1/4\kappa^2 = 2N^3/\pi^5$ and $\bar{e} = 2$. One has

$$\Delta_I^f = 2k + 6 \,, \tag{4.9a}$$

$$\Delta_I^s = 2k \,, \tag{4.9b}$$

$$\Delta_I^t = 2k + 12 \,. \tag{4.9c}$$

Set

$$\phi(k) = 4 \left[\frac{(2k+1)!}{(2k+2)!\,(2k+5)!} \right]^{1/2} , \tag{4.10a}$$

$$\sigma(k) = \Big[(2k-2)! \Big]^{-1/2} , \tag{4.10b}$$

$$\tau(k) = 16 \left[\frac{(2k+1)!\,(2k+4)!\,(2k+7)!}{(2k+6)!\,(2k+8)!\,(2k+9)!\,(2k+11)!} \right]^{1/2} . \tag{4.10c}$$

Then,

$$\bar{R}^{fff}_{I_1I_2I_3} = \frac{2}{4(\pi N)^{3/2}}\,\phi(k_1)\,\phi(k_2)\,\phi(k_3)\,2^{2\alpha}\,\frac{\Gamma(2\alpha+4)\Gamma(2\alpha+6)}{\Gamma(2\alpha+3)\Gamma(2\alpha+5)}\,\Gamma(\alpha+3)\,\times$$
$$\times\,\frac{\Gamma(2\alpha_1+3)\,\Gamma(2\alpha_2+3)\,\Gamma(2\alpha_3+3)}{\Gamma(2\alpha_1+2)\,\Gamma(2\alpha_2+2)\,\Gamma(2\alpha_3+2)}\,\Gamma\left(\alpha_1+\frac{3}{2}\right)\Gamma\left(\alpha_2+\frac{3}{2}\right)\Gamma\left(\alpha_3+\frac{3}{2}\right), \tag{4.11a}$$

$$\bar{R}^{ffs}_{I_1I_2I_3} = \frac{1}{4(\pi N)^{3/2}}\,\phi(k_1)\,\phi(k_2)\,\sigma(k_3)\,2^{2\alpha}\,\Gamma(\alpha+2)\,\times$$
$$\times\,\frac{\Gamma(2\alpha_3+6)}{\Gamma(2\alpha_3+2)}\,\Gamma\left(\alpha_1+\frac{1}{2}\right)\Gamma\left(\alpha_2+\frac{1}{2}\right)\Gamma\left(\alpha_3+\frac{5}{2}\right), \tag{4.11b}$$

$$\bar{R}^{fft}_{I_1I_2I_3} = \frac{1}{4(\pi N)^{3/2}}\,\phi(k_1)\,\phi(k_2)\,\tau(k_3)\,2^{2\alpha}\,\frac{\Gamma(2\alpha+9)}{\Gamma(2\alpha+5)}\,\Gamma(\alpha+4)\,\times$$
$$\times\,\frac{\Gamma(2\alpha_1+6)\,\Gamma(2\alpha_2+6)}{\Gamma(2\alpha_1+2)\,\Gamma(2\alpha_2+2)}\,\Gamma\left(\alpha_1+\frac{5}{2}\right)\Gamma\left(\alpha_2+\frac{5}{2}\right)\Gamma\left(\alpha_3+\frac{1}{2}\right), \tag{4.11c}$$

$$\bar{R}^{fss}_{I_1I_2I_3} = \frac{1}{4(\pi N)^{3/2}}\,\phi(k_1)\,\sigma(k_2)\,\sigma(k_3)\,2^{2\alpha}\,\Gamma(\alpha+1)\,\times$$
$$\times\,\frac{\Gamma(2\alpha_2+3)\,\Gamma(2\alpha_3+3)}{\Gamma(2\alpha_2+2)\,\Gamma(2\alpha_3+2)}\,\Gamma\left(\alpha_1-\frac{1}{2}\right)\Gamma\left(\alpha_2+\frac{3}{2}\right)\Gamma\left(\alpha_3+\frac{3}{2}\right), \tag{4.11d}$$

$$\bar{R}^{fst}_{I_1I_2I_3} = \frac{1}{4(\pi N)^{3/2}}\,\phi(k_1)\,\sigma(k_2)\,\tau(k_3)\,2^{2\alpha}\,\frac{\Gamma(2\alpha+4)\,\Gamma(2\alpha+6)}{\Gamma(2\alpha+3)\,\Gamma(2\alpha+5)}\,\Gamma(\alpha+3)\,\times$$
$$\times\,\frac{\Gamma(2\alpha_1+1)\,\Gamma(2\alpha_1+3)}{\Gamma(2\alpha_1)\,\Gamma(2\alpha_1+2)}\,\frac{\Gamma(2\alpha_2+3)\,\Gamma(2\alpha_2+7)\,\Gamma(2\alpha_2+9)}{\Gamma(2\alpha_2+2)\,\Gamma(2\alpha_2+4)\,\Gamma(2\alpha_2+6)}\,\frac{\Gamma(2\alpha_3)}{\Gamma(2\alpha_3+1)}\,\times$$
$$\times\,\Gamma\left(\alpha_1+\frac{3}{2}\right)\Gamma\left(\alpha_2+\frac{7}{2}\right)\Gamma\left(\alpha_3-\frac{1}{2}\right), \tag{4.11e}$$

$$\bar{R}^{ftt}_{I_1I_2I_3} = \frac{1}{4(\pi N)^{3/2}}\,\phi(k_1)\,\tau(k_2)\,\tau(k_3)\,\times$$
$$\times\,2^{2\alpha}\,\frac{\Gamma(2\alpha+4)\,\Gamma(2\alpha+6)\,\Gamma(2\alpha+10)\,\Gamma(2\alpha+12)}{\Gamma(2\alpha+3)\,\Gamma(2\alpha+5)\,\Gamma(2\alpha+7)\,\Gamma(2\alpha+9)}\,\Gamma(\alpha+5)\,\times$$
$$\times\,\frac{\Gamma(2\alpha_1+1)\,\Gamma(2\alpha_1+3)\,\Gamma(2\alpha_1+7)\,\Gamma(2\alpha_1+9)}{\Gamma(2\alpha_1)\,\Gamma(2\alpha_1+2)\,\Gamma(2\alpha_1+4)\,\Gamma(2\alpha_1+6)}\,\frac{\Gamma(2\alpha_2+3)}{\Gamma(2\alpha_2+2)}\,\frac{\Gamma(2\alpha_3+3)}{\Gamma(2\alpha_3+2)}\,\times$$
$$\times\,\Gamma\left(\alpha_1+\frac{7}{2}\right)\Gamma\left(\alpha_2+\frac{3}{2}\right)\Gamma\left(\alpha_3+\frac{3}{2}\right), \tag{4.11f}$$

$$\bar{R}^{sss}_{I_1I_2I_3} = \frac{1}{4(\pi N)^{3/2}}\,\sigma(k_1)\,\sigma(k_2)\,\sigma(k_3)\,2^{2\alpha}\,\Gamma(\alpha)\Gamma\left(\alpha_1+\frac{1}{2}\right)\,\times$$
$$\times\,\Gamma\left(\alpha_2+\frac{1}{2}\right)\Gamma\left(\alpha_3+\frac{1}{2}\right), \tag{4.11g}$$

$$\bar{R}^{sst}_{I_1I_2I_3} = \frac{1}{4(\pi N)^{3/2}}\,\sigma(k_1)\,\sigma(k_2)\,\tau(k_3)\,2^{2\alpha}\,\Gamma(\alpha+2)\,\frac{\Gamma(2\alpha_1+6)\,\Gamma(2\alpha_2+6)}{\Gamma(2\alpha_1+2)\,\Gamma(2\alpha_2+2)}\times$$
$$\times\,\Gamma\left(\alpha_1+\frac{5}{2}\right)\Gamma\left(\alpha_2+\frac{5}{2}\right)\Gamma\left(\alpha_3-\frac{3}{2}\right), \tag{4.11h}$$

$$\bar{R}^{tts}_{I_1I_2I_3} = \frac{1}{4(\pi N)^{3/2}}\,\tau(k_1)\,\tau(k_2)\,\sigma(k_3)\,2^{2\alpha}\,\frac{\Gamma(2\alpha+9)}{\Gamma(2\alpha+5)}\,\Gamma(\alpha+4)\times$$
$$\times\,\frac{\Gamma(2\alpha_3+10)\,\Gamma(2\alpha_3+12)}{\Gamma(2\alpha_3+2)\,\Gamma(2\alpha_3+8)}\,\Gamma\left(\alpha_1+\frac{1}{2}\right)\Gamma\left(\alpha_2+\frac{1}{2}\right)\Gamma\left(\alpha_3+\frac{9}{2}\right), \tag{4.11i}$$

$$\bar{R}^{ttt}_{I_1I_2I_3} = \frac{1}{4(\pi N)^{3/2}}\,\tau(k_1)\,\tau(k_2)\,\tau(k_3)\,2^{2\alpha}\,\frac{\Gamma(2\alpha+13)\,\Gamma(2\alpha+15)}{\Gamma(2\alpha+5)\,\Gamma(2\alpha+11)}\,\Gamma(\alpha+6)\times \tag{4.11j}$$
$$\times\,\frac{\Gamma(2\alpha_1+6)\,\Gamma(2\alpha_2+6)\,\Gamma(2\alpha_3+6)}{\Gamma(2\alpha_1+2)\,\Gamma(2\alpha_2+2)\,\Gamma(2\alpha_3+2)}\,\Gamma\left(\alpha_1+\frac{5}{2}\right)\Gamma\left(\alpha_2+\frac{5}{2}\right)\Gamma\left(\alpha_3+\frac{5}{2}\right).$$

$AdS_4\times S_7$. In this case one has $1/4\kappa^2 = N^{3/2}/2^{19/2}\,\pi^5$ and $\bar{e}=1/2$. One has

$$\Delta^f_I = \frac{1}{2}\,k+3\,, \tag{4.12a}$$
$$\Delta^s_I = \frac{1}{2}\,k\,, \tag{4.12b}$$
$$\Delta^t_I = \frac{1}{2}\,k+6\,. \tag{4.12c}$$

Set

$$\phi(k) = \frac{1}{2}\left[\frac{k!\,(k+3)!}{(k+4)!}\right]^{1/2}, \tag{4.13a}$$
$$\sigma(k) = \Big[(k+1)!\Big]^{1/2}, \tag{4.13b}$$
$$\tau(k) = \frac{1}{4}\left[\frac{k!\,(k+2)!\,(k+3)!\,(k+5)!}{(k+4)!\,(k+7)!\,(k+10)!}\right]^{1/2}. \tag{4.13c}$$

Then,

$$\bar{R}^{fff}_{I_1I_2I_3} = \frac{\pi}{2}\left(\frac{2}{N}\right)^{3/4}\,\phi(k_1)\,\phi(k_2)\,\phi(k_3)\,2^{-\alpha}\,\frac{\Gamma(\alpha+5)}{\Gamma(\alpha+3)}\,\frac{1}{\Gamma\left(\frac{1}{2}\,\alpha+\frac{5}{2}\right)}\times \tag{4.14a}$$
$$\times\,\frac{\Gamma(\alpha_1+2)\,\Gamma(\alpha_2+2)\,\Gamma(\alpha_3+2)}{\Gamma(\alpha_1+1)\,\Gamma(\alpha_2+1)\,\Gamma(\alpha_3+1)}\,\frac{1}{\Gamma\left(\frac{1}{2}\,\alpha_1+1\right)\Gamma\left(\frac{1}{2}\,\alpha_2+1\right)\Gamma\left(\frac{1}{2}\,\alpha_3+1\right)},$$

$$\bar{R}^{ffs}_{I_1I_2I_3} = \frac{\pi}{2}\left(\frac{2}{N}\right)^{3/4}\,\phi(k_1)\,\phi(k_2)\,\sigma(k_3)\,2^{-\alpha}\,\frac{1}{\Gamma\left(\frac{1}{2}\alpha+2\right)}\,\frac{\Gamma(\alpha_3+5)}{\Gamma(\alpha_3+1)}\times$$
$$\times\,\frac{1}{\Gamma\left(\frac{1}{2}\,(\alpha_1+1)\right)\Gamma\left(\frac{1}{2}\,(\alpha_2+1)\right)\Gamma\left(\frac{1}{2}\,(\alpha_3+3)\right)}, \tag{4.14b}$$

$$\bar{R}^{fft}_{I_1 I_2 I_3} = \frac{\pi}{2} \left(\frac{2}{N}\right)^{3/4} \phi(k_1)\,\phi(k_2)\,\tau(k_3)\,2^{-\alpha}\,\frac{\Gamma(\alpha+8)}{\Gamma(\alpha+4)}\,\frac{1}{\Gamma\left(\frac{1}{2}\,\alpha+3\right)} \times \tag{4.14c}$$

$$\times \frac{\Gamma(\alpha_1+5)\,\Gamma(\alpha_2+5)}{\Gamma(\alpha_1+1)\,\Gamma(\alpha_2+1)}\,\frac{1}{\Gamma\left(\frac{1}{2}\,(\alpha_1+3)\right)\Gamma\left(\frac{1}{2}\,(\alpha_2+3)\right)\Gamma\left(\frac{1}{2}\,(\alpha_3+1)\right)}\,,$$

$$\bar{R}^{fss}_{I_1 I_2 I_3} = \frac{\pi}{2} \left(\frac{2}{N}\right)^{3/4} \phi(k_1)\,\sigma(k_2)\,\sigma(k_3)\,2^{-\alpha}\,\frac{1}{\Gamma\left(\frac{1}{2}\,\alpha+\frac{3}{2}\right)} \times$$

$$\times \frac{\Gamma(\alpha_2+2)\,\Gamma(\alpha_3+2)}{\Gamma(\alpha_2+1)\,\Gamma(\alpha_3+1)}\,\frac{1}{\Gamma\left(\frac{1}{2}\,\alpha_1\right)\Gamma\left(\frac{1}{2}\,\alpha_2+1\right)\Gamma\left(\frac{1}{2}\,\alpha_3+1\right)}\,, \tag{4.14d}$$

$$\bar{R}^{fst}_{I_1 I_2 I_3} = \frac{\pi}{2} \left(\frac{2}{N}\right)^{3/4} \phi(k_1)\,\sigma(k_2)\,\tau(k_3)\,2^{-\alpha}\,\frac{\Gamma(\alpha+5)}{\Gamma(\alpha+3)}\,\frac{1}{\Gamma(\,\frac{1}{2}\,\alpha+\frac{5}{2}\,)}\,\frac{\Gamma(\alpha_1+2)}{\Gamma(\alpha_1)} \times$$

$$\times \frac{\Gamma(\alpha_2+8)}{\Gamma(\alpha_2+1)}\,\frac{\Gamma(\alpha_3)}{\Gamma(\alpha_3+1)}\,\frac{1}{\Gamma\left(\frac{1}{2}\,\alpha_1+1\right)\Gamma\left(\frac{1}{2}\,\alpha_2+2\right)\Gamma\left(\frac{1}{2}\,\alpha_3\right)}\,, \tag{4.14e}$$

$$\bar{R}^{ftt}_{I_1 I_2 I_3} = \frac{\pi}{2} \left(\frac{2}{N}\right)^{3/4} \phi(k_1)\,\tau(k_2)\,\tau(k_3)\,2^{-\alpha}\,\frac{\Gamma(\alpha+11)}{\Gamma(\alpha+3)}\,\frac{1}{\Gamma\left(\frac{1}{2}\,\alpha+\frac{7}{2}\right)}\,\frac{\Gamma(\alpha_1+8)}{\Gamma(\alpha_1)} \times$$

$$\times \frac{\Gamma(\alpha_2+2)}{\Gamma(\alpha_2+1)}\,\frac{\Gamma(\alpha_3+2)}{\Gamma(\alpha_3+1)}\,\frac{1}{\Gamma\left(\frac{1}{2}\,\alpha_1+2\right)\Gamma\left(\frac{1}{2}\,\alpha_2+1\right)\Gamma\left(\frac{1}{2}\,\alpha_3+1\right)}\,, \tag{4.14f}$$

$$\bar{R}^{sss}_{I_1 I_2 I_3} = \frac{\pi}{2} \left(\frac{2}{N}\right)^{3/4} \sigma(k_1)\,\sigma(k_2)\,\sigma(k_3)\,2^{-\alpha}\,\frac{1}{\Gamma\left(\frac{1}{2}\,\alpha+1\right)} \times$$

$$\times \frac{1}{\Gamma\left(\frac{1}{2}\,(\alpha_1+1)\right)\Gamma\left(\frac{1}{2}\,(\alpha_2+1)\right)\Gamma\left(\frac{1}{2}\,(\alpha_3+1)\right)}\,, \tag{4.14g}$$

$$\bar{R}^{sst}_{I_1 I_2 I_3} = \frac{\pi}{2} \left(\frac{2}{N}\right)^{3/4} \sigma(k_1)\,\sigma(k_2)\,\tau(k_3)\,2^{-\alpha}\,\frac{1}{\Gamma\left(\frac{1}{2}\,\alpha+2\right)} \times \tag{4.14h}$$

$$\times \frac{\Gamma(\alpha_1+5)\,\Gamma(\alpha_2+5)}{\Gamma(\alpha_1+1)\,\Gamma(\alpha_2+1)}\,\frac{1}{\Gamma\left(\frac{1}{2}\,(\alpha_1+3)\right)\Gamma\left(\frac{1}{2}\,(\alpha_2+3)\right)\Gamma\left(\frac{1}{2}\,(\alpha_3-1)\right)}\,,$$

$$\bar{R}^{tts}_{I_1 I_2 I_3} = \frac{\pi}{2} \left(\frac{2}{N}\right)^{3/4} \tau(k_1)\,\tau(k_2)\,\sigma(k_3)\,2^{-\alpha}\,\frac{\Gamma(\alpha+8)}{\Gamma(\alpha+4)}\,\frac{1}{\Gamma\left(\frac{1}{2}\,\alpha+3\right)} \times \tag{4.14i}$$

$$\times \frac{\Gamma(\alpha_3+5)\,\Gamma(\alpha_3+11)}{\Gamma(\alpha_3+1)\,\Gamma(\alpha_3+3)}\,\frac{1}{\Gamma\left(\frac{1}{2}\,(\alpha_1+1)\right)\Gamma\left(\frac{1}{2}\,(\alpha_2+1)\right)\Gamma\left(\frac{1}{2}\,(\alpha_3+5)\right)}\,,$$

$$\bar{R}^{ttt}_{I_1 I_2 I_3} = \frac{\pi}{2} \left(\frac{2}{N}\right)^{3/4} \tau(k_1)\,\tau(k_2)\,\tau(k_3)\,2^{-\alpha}\,\frac{\Gamma(\alpha+8)\,\Gamma(\alpha+14)}{\Gamma(\alpha+4)\,\Gamma(\alpha+6)}\,\frac{1}{\Gamma\left(\frac{1}{2}\,\alpha+4\right)} \times$$

$$\times \frac{\Gamma(\alpha_1+5)\,\Gamma(\alpha_2+5)\,\Gamma(\alpha_3+5)}{\Gamma(\alpha_1+1)\,\Gamma(\alpha_2+1)\,\Gamma(\alpha_3+1)} \times$$

$$\times \frac{1}{\Gamma\left(\frac{1}{2}\,(\alpha_1+3)\right)\Gamma\left(\frac{1}{2}\,(\alpha_2+3)\right)\Gamma\left(\frac{1}{2}\,(\alpha_3+3)\right)} \tag{4.14j}$$

$AdS_5 \times S_5$**.** In this case one has $1/4\kappa^2 = N^2/8\pi^5$ and $\bar{e} = 1$. One has

$$\Delta_I^f = k + 4\,, \tag{4.15a}$$

$$\Delta_I^s = k\,, \tag{4.15b}$$

$$\Delta_I^t = k + 8\,. \tag{4.15c}$$

Set

$$\phi(k) = \left[\frac{k!}{(k+3)!}\right]^{1/2}, \tag{4.16a}$$

$$\sigma(k) = k^{1/2}\,, \tag{4.16b}$$

$$\tau(k) = \left[\frac{k!\,(k+1)!\,(k+2)!}{(k+5)!\,(k+6)!\,(k+7)!}\right]^{1/2}. \tag{4.16c}$$

Then,

$$\bar{R}^{fff}_{I_1 I_2 I_3} = \frac{1}{N}\,\phi(k_1)\,\phi(k_2)\,\phi(k_3)\,\frac{\Gamma(\alpha+4)}{\Gamma(\alpha+2)}\,\frac{\Gamma(\alpha_1+2)\,\Gamma(\alpha_2+2)\,\Gamma(\alpha_3+2)}{\Gamma(\alpha_1+1)\,\Gamma(\alpha_2+1)\,\Gamma(\alpha_3+1)}\,, \tag{4.17a}$$

$$\bar{R}^{ffs}_{I_1 I_2 I_3} = \frac{1}{N}\,\phi(k_1)\,\phi(k_2)\,\sigma(k_3)\,\frac{\Gamma(\alpha_3+3)\,\Gamma(\alpha_3+4)}{\Gamma(\alpha_3+1)\,\Gamma(\alpha_3+2)}\,, \tag{4.17b}$$

$$\begin{aligned}\bar{R}^{fft}_{I_1 I_2 I_3} &= \frac{1}{N}\,\phi(k_1)\,\phi(k_2)\,\tau(k_3)\,\frac{\Gamma(\alpha+5)\,\Gamma(\alpha+6)}{\Gamma(\alpha+3)\,\Gamma(\alpha+4)}\,\frac{\Gamma(\alpha_1+3)\,\Gamma(\alpha_1+4)}{\Gamma(\alpha_1+1)\,\Gamma(\alpha_1+2)}\,\times \\ &\quad\times\frac{\Gamma(\alpha_2+3)\,\Gamma(\alpha_2+4)}{\Gamma(\alpha_2+1)\,\Gamma(\alpha_2+2)}\,,\end{aligned} \tag{4.17c}$$

$$\bar{R}^{fss}_{I_1 I_2 I_3} = \frac{1}{N}\,\phi(k_1)\,\sigma(k_2)\,\sigma(k_3)\,\frac{\Gamma(\alpha_2+2)}{\Gamma(\alpha_2+1)}\,\frac{\Gamma(\alpha_3+2)}{\Gamma(\alpha_3+1)}\,, \tag{4.17d}$$

$$\begin{aligned}\bar{R}^{fst}_{I_1 I_2 I_3} &= \frac{1}{N}\,\phi(k_1)\,\sigma(k_2)\,\tau(k_3)\,\frac{\Gamma(\alpha+4)}{\Gamma(\alpha+2)}\,\frac{\Gamma(\alpha_1+2)}{\Gamma(\alpha_1)}\,\times \\ &\quad\times\frac{\Gamma(\alpha_2+5)\,\Gamma(\alpha_2+6)}{\Gamma(\alpha_2+1)\,\Gamma(\alpha_2+3)}\,\frac{\Gamma(\alpha_3)}{\Gamma(\alpha_3+1)}\,,\end{aligned} \tag{4.17e}$$

$$\begin{aligned}\bar{R}^{ftt}_{I_1 I_2 I_3} &= \frac{1}{N}\,\phi(k_1)\,\tau(k_2)\,\tau(k_3)\,\frac{\Gamma(\alpha+7)\,\Gamma(\alpha+8)}{\Gamma(\alpha+2)\,\Gamma(\alpha+5)}\,\frac{\Gamma(\alpha_1+5)\,\Gamma(\alpha_1+6)}{\Gamma(\alpha_1)\,\Gamma(\alpha_1+3)}\,\times \\ &\quad\times\frac{\Gamma(\alpha_2+2)}{\Gamma(\alpha_2+1)}\,\frac{\Gamma(\alpha_3+2)}{\Gamma(\alpha_3+1)}\,,\end{aligned} \tag{4.17f}$$

$$\bar{R}^{sss}_{I_1 I_2 I_3} = \frac{1}{N}\,\sigma(k_1)\,\sigma(k_2)\,\sigma(k_3)\,, \tag{4.17g}$$

$$\bar{R}^{sst}_{I_1 I_2 I_3} = \frac{1}{N}\,\sigma(k_1)\,\sigma(k_2)\,\tau(k_3)\,\frac{\Gamma(\alpha_1+3)\,\Gamma(\alpha_1+4)\Gamma(\alpha_2+3)\,\Gamma(\alpha_2+4)}{\Gamma(\alpha_1+1)\,\Gamma(\alpha_1+2)\,\Gamma(\alpha_2+1)\,\Gamma(\alpha_2+2)}\,, \tag{4.17h}$$

$$\bar{R}^{tts}_{I_1I_2I_3} = \frac{1}{N}\,\tau(k_1)\,\tau(k_2)\,\sigma(k_3)\,\frac{\Gamma(\alpha+5)\,\Gamma(\alpha+6)}{\Gamma(\alpha+3)\,\Gamma(\alpha+4)}\,\frac{\Gamma(\alpha_3+7)\,\Gamma(\alpha_3+8)}{\Gamma(\alpha_3+1)\,\Gamma(\alpha_3+2)}\,, \tag{4.17i}$$

$$\bar{R}^{ttt}_{I_1I_2I_3} = \frac{1}{N}\,\tau(k_1)\,\tau(k_2)\,\tau(k_3)\,\frac{\Gamma(\alpha+9)\,\Gamma(\alpha+10)}{\Gamma(\alpha+3)\,\Gamma(\alpha+4)}\times \tag{4.17j}$$
$$\times\frac{\Gamma(\alpha_1+3)\,\Gamma(\alpha_1+4)\,\Gamma(\alpha_2+3)\,\Gamma(\alpha_2+4)\,\Gamma(\alpha_3+3)\,\Gamma(\alpha_3+4)}{\Gamma(\alpha_1+1)\,\Gamma(\alpha_1+2)\,\Gamma(\alpha_2+1)\,\Gamma(\alpha_2+2)\,\Gamma(\alpha_3+1)\,\Gamma(\alpha_3+2)}\,.$$

It is useful to remember that the value of the integer k of the operators $\mathcal{O}^s_I$, $\mathcal{O}^f_I$, $\mathcal{O}^t_I$ belonging to the same multiplet are related as follows:

$$k^s = k\,, \qquad k^f = k-2\,, \qquad k^t = k-4\,. \tag{4.18}$$

Some of these results have been obtained by other groups as well. Namely, in the $SCFT_4$ case, the 3-point function of the chiral primary operators eq. (4.17g) was produced in the seminal paper [13], while eq. (4.17h) was contained in [17]. As for $SCFT_6$, eq. (4.11g) was calculated in [15], though that result differs for a factor from ours [23]. We have computed in previous works the $s-t$ correlations functions [23, 24], extended here above to the full universal sector.

5. Conclusions

We have computed in a systematic way the large-N limit of the 3-point functions of the scalars belonging to the universal sector which is common to all of the maximal supersymmetric CFT in $d = 3, 4, 6$. The infra-red dynamics of the M2-, D3- and M5-branes belongs to the universality classes identified by these $SCFT_{3,4,6}$, and in fact we used the AdS/CFT correspondence to compute some scalar correlation functions in the latter. That constitutes the main result of this paper.

At large N, all the operators group into short (also called chiral) supersymmetric multiplets [31]–[35]. In particular, the operators $\mathcal{O}^s$ are the chiral primaries which generate all of the other by application of the superconformal algebra [36, 37]. In $d = 4$ it was shown in [38] by using a superspace approach that the 3-point function of all chiral operators are fixed by a unique superspace conformal invariant up to a single coefficient, which can be easily read off from $\langle\mathcal{O}^s\mathcal{O}^s\mathcal{O}^s\rangle$. Thus, it should be possible to obtain the correlation functions of the descendants, including the ones listed in section 4, by such superspace techniques, though this may be laborious. A similar strategy has not been worked out for the $SCFT_{3,6}$, to our knowledge, and it is not known how many independent numbers describe the correlation functions of the chiral multiplets. Investigation of the stress tensor 3-point functions (belonging to the family identified by the chiral primary with $k = 2$) show only a single overall coefficient for all of the $SCFT_{3,6}$ [39], and this may suggest that also in these new cases

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supersymmetry fixes the correlations function of chiral multiplets up to a constant. It would indeed be interesting to develop a superspace approach and reproduce the descendant correlation functions from the chiral ones. More importantly, it would be nice to investigate their $1/N$ corrections. On the other hand, the general model we used to obtain the scalar 3-point function can still be employed to compute the 4-point functions of the chiral primaries in a systematic way for the $SCFT_{3,4,6}$. In the light of some results for the super Yang-Mills case [20], that might be a formidable task but still within the reach of future research.

A. Spherical harmonics

We describe the n-dimensional sphere S_n of radius ρ as $S_n = \{x \in \mathbb{R}^{n+1} \mid x^2 = \rho^2\}$.

Since $S_n \subset \mathbb{R}^{n+1}$, we can represent tensor fields on S_n by means of tensor fields on $\mathbb{R}^{n+1}$ of the same type satisfying certain conditions. Since further S_n is naturally equipped with the metric induced by the euclidean metric of $\mathbb{R}^{n+1}$, we need not distinguish covariant and contravariant tensor indices. Specifically, a rank s tensor field $T_{\alpha_1\cdots\alpha_s}$ on S_n may be viewed as a rank s tensor field $T_{i_1\cdots i_s}$ defined on a neighborhood of $S_n \subset \mathbb{R}^{n+1}$ with no components normal to S_n:

$$x_i\, T_{i_1\cdots i_{a-1} i i_{a+1}\cdots i_s}(x) = 0\,, \quad 1 \le a \le s\,. \tag{A.1}$$

In particular, the induced metric $g_{\alpha\beta}$ of S_n is represented by

$$g_{ij}(x) = \delta_{ij} - \frac{x_i\, x_j}{r^2}\,, \tag{A.2}$$

where $r = (x_i x_i)^{1/2}$. Therefore, the contraction of two rank s tensor fields $T_{\alpha_1\cdots\alpha_s}$, $U_{\alpha_1\cdots\alpha_s}$ is given by

$$T^{\alpha_1\cdots\alpha_s}\, U_{\alpha_1\cdots\alpha_s} = T_{i_1\cdots i_s}\, U_{i_1\cdots i_s}\,. \tag{A.3}$$

For a rank s tensor field $T_{\alpha_1\cdots\alpha_s}$, the covariant derivative $\nabla_\alpha T_{\alpha_1\cdots\alpha_s}$ is represented by

$$\nabla_i T_{i_1\cdots i_s}(x) = \left(\partial_i - \frac{x_i}{r}\,\partial_r\right) T_{i_1\cdots i_s}(x) + \sum_{a=1}^{s} \frac{x_{i_a}}{r^2}\, T_{i_1\cdots i_{a-1} i i_{a+1}\cdots i_s}(x)\,, \tag{A.4}$$

where $\partial_r = \partial/\partial r$.

For any function F on S_n, the integral of F on S_n is given by

$$\int_{S_n} d^n \operatorname{vol} F = \int_{S_n} r^n\, d\Omega_n(x)\, F(x)\Big|_{r=\rho}\,, \tag{A.5}$$

where $d\Omega_n(x)$ is the standard n dimensional volume on S_n defined as usual by $d^{n+1}x = r^n dr d\Omega_n(x)$.

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Using the above integration formula, one can provide the space of rank s tensor fields with a Hilbert space structure in obvious fashion.

Consider the rank s tensor fields

$$Y^{(s)}_{Ii_1\cdots i_s}(x) = \mathcal{Y}^{(s)}_{Ii_1\cdots i_s;j_1\cdots j_k}\, r^{-k}\, x_{j_1}\cdots x_{j_k}\,, \tag{A.6}$$

where the $\mathcal{Y}^{(s)}_{Ii_1\cdots i_s;j_1\cdots j_k}$ are constants satisfying the following conditions:

(i) $\mathcal{Y}^{(s)}_{Ii_1\cdots i_s;j_1\cdots j_k}$ is symmetric and traceless in $i_1,\ldots,i_s$;

(ii) $\mathcal{Y}^{(s)}_{Ii_1\cdots i_s;j_1\cdots j_k}$ is symmetric and traceless in $j_1,\ldots,j_k$;

(iii) the following relation holds:

$$\mathcal{Y}^{(s)}_{Ii_1\cdots i_{s-1}\{i_s;j_1\cdots j_k\}} = 0\,, \tag{A.7}$$

where $\{\cdots\}$ denotes total symmetrization.

The index I labels the different choices of the constants $\mathcal{Y}^{(s)}_{Ii_1\cdots i_s;j_1\cdots j_k}$. Note that $Y^{(s)}_{Ii_1\cdots i_s}$ is characterized by the non-negative integer k, which may be thought of as a function of the label I. It can be shown that, if $Y^{(s)}_{Ii_1\cdots i_s} \neq 0$, then necessarily $k \geq s$.

Then, the $Y^{(s)}_{Ii_1\ldots i_s}$ are symmetric traceless rank s tensor fields on S_n. Further, they are divergenceless (transversal):

$$\nabla_i\, Y^{(s)}_{Iii_2\cdots i_s} = 0\,. \tag{A.8}$$

Finally, the $Y^{(s)}_{Ii_1\cdots i_s}$ are eigenfields of the laplacian:

$$-\nabla_i\nabla_i Y^{(s)}_{Ii_1\cdots i_s} = \frac{1}{r^2}\Big[k(k+n-1)-s\Big]Y^{(s)}_{Ii_1\cdots i_s}\,. \tag{A.9}$$

If, for fixed s, we choose a maximal set of constants $\mathcal{Y}^{(s)}_{Ii_1\cdots i_s;j_1\cdots j_k}$ such that

$$\mathcal{Y}^{(s)}_{Ii_1\cdots i_s;j_1\cdots j_k}\,\mathcal{Y}^{(s)}_{Ji_1\cdots i_s;j_1\cdots j_k} = \delta_{IJ} \tag{A.10}$$

for all k, then, the tensor fields $\{Y^{(s)}_{Ii_1\cdots i_s} \mid I\}$ provide an orthogonal basis of the space of divergenceless symmetric traceless rank s tensor fields on S_n.

In concrete applications, one has to compute the integrals of scalars formed by contraction of several tensor spherical harmonics $Y^{(s_\ell)}$ and their covariant derivatives. This can be done by a systematic use of the formula

$$\int_{S^n} r^n\, d\Omega_n(x)\, r^{-2m}\, x^{i_1}\cdots x^{i_{2m}}\Bigg|_{r=\rho} = \rho^n\,\omega_n\,\frac{(n-1)!!}{(2m+n-1)!!}\times \\ \times\,(\text{all possible Wick contractions})\,, \tag{A.11}$$

where "all possible Wick contractions" are given by the sum of $(2m-1)!!$ terms obtained by using $\langle r^{-2}x^i x^j\rangle = \delta^{ij}$ as elementary Wick contraction and ω_n is the volume of the unit sphere

$$\omega_n = \frac{2\pi^{\frac{n+1}{2}}}{\Gamma\left(\frac{n+1}{2}\right)} \,. \tag{A.12}$$

The results are always expressible in terms of suitable contractions of the coefficients $\mathcal{Y}^{(s_\ell)}_{Ii_1\cdots i_{s_\ell};j_1\cdots j_{k_\ell}}$ generically denoted as $\langle\prod_\ell \mathcal{Y}^{(s_\ell)}_{I_\ell}\rangle$ and certain numerical functions of the k_ℓ.

For two and three tensor spherical harmonics, the only such functions are

$$z_I = \omega_n \, \frac{(n-1)!!\, k!}{(2k+n-1)!!} \,, \tag{A.13}$$

$$a_{I_1 I_2 I_3} = \omega_n \, \frac{(n-1)!!}{(2\alpha+n-1)!!} \, \frac{k_1!\, k_2!\, k_3!}{\alpha_1!\, \alpha_2!\, \alpha_3!} \,, \tag{A.14}$$

where

$$\begin{aligned}
\alpha_1 &= \frac{1}{2}\,(k_2+k_3-k_1)\,, \\
\alpha_2 &= \frac{1}{2}\,(k_3+k_1-k_2)\,, \\
\alpha_3 &= \frac{1}{2}\,(k_1+k_2-k_3)\,, && \text{(A.15a)}\\
\alpha &= \frac{1}{2}\,(k_1+k_2+k_3)\,. && \text{(A.15b)}
\end{aligned}$$

In this paper, $\rho = \bar{e}^{-1}$. Further, we consider only $s = 0, 2$. We set $\mathcal{C}_{I;j_1\cdots j_k} = \mathcal{Y}^{(0)}_{I;j_1\cdots j_k}$ and $\mathcal{T}_{Ii_1i_2;j_1\cdots j_k} = \mathcal{Y}^{(2)}_{Ii_1i_2;j_1\cdots j_k}$.

The main contractions are the following. $\langle\mathcal{C}_{I_1}\mathcal{C}_{I_2}\,\mathcal{C}_{I_3}\rangle$ denotes the unique SO$(n+1)$ scalar contraction of three tensors $\mathcal{C}_{Ii_1\cdots i_k}$ and in a similar fashion we define

$$\langle\mathcal{T}_{I_1}\,\mathcal{C}_{I_2}\,\mathcal{C}_{I_3}\rangle = \langle\mathcal{T}_{I_1ab}\,\mathcal{C}_{I_2;a}\,\mathcal{C}_{I_3;b}\rangle\,, \tag{A.16a}$$

$$\langle\mathcal{T}_{I_1}\,\mathcal{T}_{I_2}\,\mathcal{C}_{I_3}\rangle = \langle\mathcal{T}_{I_1ab}\,\mathcal{T}_{I_2ab}\,\mathcal{C}_{I_3}\rangle\,, \tag{A.16b}$$

$$\begin{aligned}
\langle\mathcal{T}_{I_1}\,\mathcal{T}_{I_2}\,\mathcal{T}_{I_3}\rangle = {} & 4\,\langle\mathcal{T}_{I_1ab}\,\mathcal{T}_{I_2bc}\,\mathcal{T}_{I_3ca}\rangle + \\
& + \sum_{\text{c.p.}123} \alpha_1\Big(2\,\langle\mathcal{T}_{I_1ab}\,\mathcal{T}_{I_2ac;d}\,\mathcal{T}_{I_3bd;c}\rangle + \langle\mathcal{T}_{I_1ab}\,\mathcal{T}_{I_2cd;a}\,\mathcal{T}_{I_3cd;b}\rangle\Big)\,,
\end{aligned} \tag{A.16c}$$

where on the right-hand side one takes the unique contraction of the hidden indices, and where the notation c.p. 123 stands for cyclic permutations of 123.

B. Auxiliary functions

Here is the complete list of the auxiliary functions F appearing in (3.4a)–(3.4j).

$$F^{fss} = (D-2)\,\alpha_2\,\alpha_3\,, \tag{B.1a}$$

$$F^{ftt} = (D-2)\,\alpha_2\,\alpha_3\,, \tag{B.1b}$$

$$F^{fst} = (D-2)\,\alpha_1(\alpha+1+p)\,, \tag{B.1c}$$

$$\begin{aligned}
F^{sss} &= (1+p)(D-3-p)\Big({\alpha_1}^2\,\alpha_2+{\alpha_2}^2\,\alpha_1+{\alpha_2}^2\,\alpha_3+{\alpha_3}^2\,\alpha_2+{\alpha_3}^2\,\alpha_1+{\alpha_1}^2\,\alpha_3\Big)+\\
&\quad+\Big(3D-8+(2D-8)p-2p^2\Big)\,\alpha_1\,\alpha_2\,\alpha_3+\\
&\quad+(1+p)\left(-\frac{1}{2}\,D+2+p\right)\Big(\alpha_1\,\alpha_2+\alpha_2\,\alpha_3+\alpha_3\,\alpha_1\Big)\,,
\end{aligned} \tag{B.1d}$$

$$\begin{aligned}
F^{sst} &= (1+p)(D-3-p)\Big({\alpha_1}^2\,\alpha_3+{\alpha_3}^2\,\alpha_1+{\alpha_2}^2\,\alpha_3+{\alpha_3}^2\,\alpha_2\Big)+\\
&\quad+\Big(-1+(D-4)\,p-p^2\Big)\,\big({\alpha_1}^2\,\alpha_2+{\alpha_2}^2\,\alpha_1\big)+\\
&\quad+\Big(D-4+(2D-8)\,p-2p^2\Big)\,\alpha_1\,\alpha_2\,\alpha_3+\\
&\quad+(1+p)\,p(D-3-p)\,(\alpha_1\,\alpha_3+\alpha_2\,\alpha_3)+\\
&\quad+(1+p)\Big(-D+2+(D-3)\,p-p^2\Big)\,\alpha_1\,\alpha_2+\\
&\quad+(1+p)(D-3-p)\Big(-{\alpha_1}^2-{\alpha_2}^2+(1+p){\alpha_3}^2\Big)+\\
&\quad+(1+p)^2(-D+3+p)\,(\alpha_1+\alpha_2+\alpha_3)\,,
\end{aligned} \tag{B.1e}$$

$$\begin{aligned}
F^{tts} &= (1+p)(D-3-p)\Big({\alpha_1}^2\,\alpha_3+{\alpha_3}^2\,\alpha_1+{\alpha_2}^2\,\alpha_3+{\alpha_3}^2\,\alpha_2\Big)+\\
&\quad+\Big(-1+(D-4)\,p-p^2\Big)\,\big({\alpha_1}^2\,\alpha_2+{\alpha_2}^2\,\alpha_1\big)+\\
&\quad+\Big(D-4+(2D-8)\,p-2p^2\Big)\,\alpha_1\,\alpha_2\,\alpha_3+\\
&\quad+(1+p)\left(\frac{5}{2}\,D-8+(2D-9)\,p-2p^2\right)(\alpha_1\,\alpha_3+\alpha_2\,\alpha_3)+\\
&\quad+(1+p)\left(\frac{3}{2}\,D-6+(2D-9)\,p-2p^2\right)\alpha_1\,\alpha_2+\\
&\quad+(1+p)\left(\frac{3}{2}\,D-5+(D-5)\,p-p^2\right)\big({\alpha_1}^2+{\alpha_2}^2\big)+\\
&\quad+(1+p)^2\left(\frac{3}{2}\,D-5+(D-5)\,p-p^2\right)(\alpha_1+\alpha_2)\,,
\end{aligned} \tag{B.1f}$$

$$\begin{aligned}
F^{ttt} &= (1+p)(D-3-p)\Big({\alpha_1}^2\,\alpha_2+{\alpha_2}^2\,\alpha_1+{\alpha_2}^2\,\alpha_3+{\alpha_3}^2\,\alpha_2+{\alpha_3}^2\,\alpha_1+{\alpha_1}^2\,\alpha_3\Big)+\\
&\quad+\Big(3D-8+(2D-8)\,p-2p^2\Big)\,\alpha_1\,\alpha_2\,\alpha_3+
\end{aligned}$$

$$
\begin{aligned}
&+ (1+p)\Big(4D - 13 + (3D-14)\,p - 3p^2\Big)\Big(\alpha_1\,\alpha_2 + \alpha_2\,\alpha_3 + \alpha_3\,\alpha_1\Big) + \\
&+ (1+p)^2\left(D - \frac{7}{2} - p\right)\left({\alpha_1}^2 + {\alpha_2}^2 + {\alpha_3}^2\right) + \\
&+ (1+p)^2\left(3D - 11 + \left(2D - \frac{21}{2}\right)p - 2p^2\right)(\alpha_1 + \alpha_2 + \alpha_3) + \\
&+ (2+p)(1+p)^3(D-4-p)\,. \qquad \text{(B.1g)}
\end{aligned}
$$

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